Lesson 8-3: Proving Triangles Similar

Similar triangles…almost but not quite congruent

Yesterday we learned the basics of similarity: polygons are considered similar when:

1. All corresponding angles are congruent.
2. The lengths of all corresponding sides are proportional.

Today we are going to work specifically with triangles. If you recall, there are many ways to determine if triangles are congruent. You can think of similar triangles as triangles that are almost, but not quite congruent.

What does this have to do with anything?

This stuff sure seems abstract. What application does it have? Is this stuff ever even used in the real world?

Yesterday we learned about the golden rectangle. It is a rectangle with a specific length to width ratio. This ratio (the golden ratio) is found to be pleasing to the eye for most people. The same holds true for similar triangles (or other polygons for that matter).

One way to think about similar triangles is two triangles that are not identical but do balance each other nicely. Thus, you will see similar triangles used extensively in art and architecture. Balance and proportionality is **very** important to the human eye…it is a fundamental element of beauty. So, being able to work with and come up with similar shapes is central to the arts and architecture.

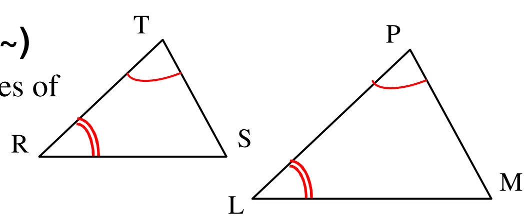
Three ways to determine triangle similarity

There are three basic ways to determine if two triangles are similar. They are:

Postulate 8-1 Angle-Angle Similarity (AA~)

If two angles of a triangle are congruent to two angles of another triangle, then the triangles are similar.

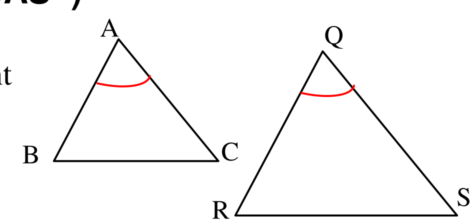
$$\text{If } \angle T \cong \angle P \;\&\; \angle R \cong \angle L \text{ then } \Delta TRS \sim \Delta PLM$$



Theorem 8-1 Side-Angle-Side Similarity (SAS~)

If an angle of one triangle is congruent to an angle of another triangle, **and** the sides including the congruent angles are proportional, then the triangles are similar.

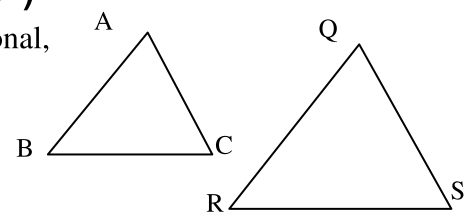
$$\text{If } \angle A \cong \angle Q \;\&\; \frac{AB}{QR} = \frac{AC}{QS} \text{ then } \Delta ABC \sim \Delta QRS$$



Theorem 8-2 Side-Side-Side Similarity (SSS~)

If the corresponding sides of two triangles are proportional, then the triangles are similar.

$$\text{If } \frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS} \text{ then } \Delta ABC \sim \Delta QRS$$



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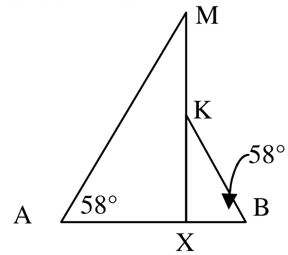
Triangle similarity examples

1. $\overline{MX} \perp \overline{AB}$. Explain why the triangles are similar. Write a similarity statement.



We have angle info & no length info so look at AA~ & SAS~.

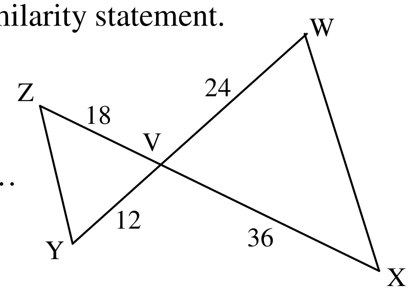
$\angle AXM$ & $\angle BXK$ are right $\angle 's$ and therefore are $\cong$.

$m\angle A = m\angle B = 58, so\ \angle A \cong \angle B$

We have two $\cong \angle 's$ so we're done.

Therefore $\Delta AXM \sim \Delta BXK$ by AA~ Postulate.

2. Explain why the triangles must be similar. Write a similarity statement.



Look for either AA~, SAS~ or SSS~.

$\angle ZVY \cong \angle WVX$ (vertical angles)

Hmm, this is the only angle set we can work with…

$\frac{VY}{VW} = \frac{12}{24} = \frac{1}{2}$ & $\frac{VZ}{VX} = \frac{18}{36} = \frac{1}{2}$ so $\frac{VY}{VW} = \frac{VZ}{VX}$

So $\Delta ZVY \sim \Delta XVW$ by SAS~ Theorem

3. $ABCD$ is a parallelogram. Find WY.

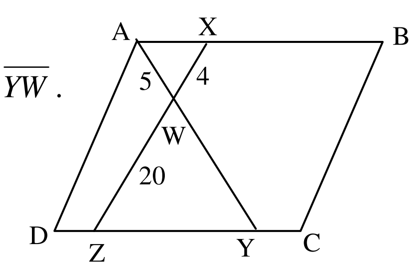
We have a lot of side length info so let's look for SAS~ or SSS~. We don't have the lengths of all three sides so work on SAS~.

We first need to determine corresponding sides.

Since it's a parallelogram, $\overline{AB} \parallel \overline{DC}$ with $\overline{AY}$ & $\overline{XZ}$ forming transversals.

$\angle AXW \cong \angle YZW$ & $\angle WAX \cong \angle WYZ$ (alt. int. $\angle s$).

So $\Delta AXW \sim \Delta YZW$ by AA~ Postulate.



Thus, $\overline{XW}$ would correspond with $\overline{ZW}$ and $\overline{AW}$ with $\overline{YW}$.

We then have

$$\frac{AW}{WY} = \frac{XW}{ZW} \ \& \ \frac{AW}{WY} = \frac{5}{WY} \ \& \ \frac{XW}{ZW} = \frac{4}{20} = \frac{1}{5}$$

So $\frac{5}{WY} = \frac{1}{5}$; $WY = 5 \bullet 5 = 25$

A practical application…indirect measurement

You can use similar triangles to determine measurements that are difficult to obtain directly. This is called **indirect measurement**. A practical application of this uses the fact that light reflects off a mirror at the same angle that it hits the mirror at.

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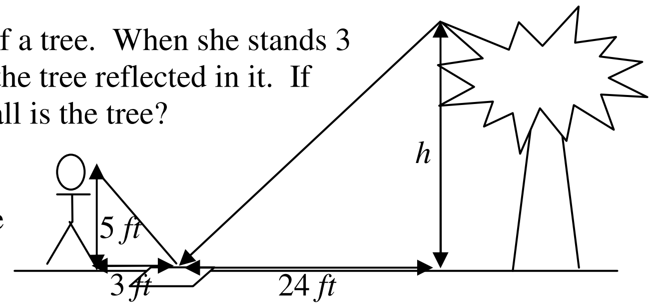
Indirect measurement example

4. Joan places a mirror 24 *ft* from the base of a tree. When she stands 3 *ft* from the mirror, she can see the top of the tree reflected in it. If her eyes are 5 *ft* above the ground, how tall is the tree?



First look for AA~, SAS~ or SSS~.

We have two right triangles. Also the angles formed at the mirror are congruent (light reflection). Thus we have two congruent angles so the two triangles are similar by the AA~ Postulate.

The distance to the mirror legs correspond; the height legs correspond. Thus we have the following proportional statement:

$$\frac{5}{3} = \frac{h}{24};\ 3h = 5 \bullet 24;\ h = \frac{120}{3} = 40\,ft$$

Homework Assignment

Pg 435 - #1-19, 22-28, 30-39, 53-57
Pg 429 - #1-10